Control Lec 4

Bode diagram

Stability

① Absloute stability

- Routh

Relative stability

* Bode diagram
* Polar plot
* Nyquist

→ Relative stability: To what range or to what degree the system is stable through stability indicators [phase margin (PM) and gain margin (GM)]

Frequency analysis

→ given T.F in s-domain & we want to describe it in Freq. domain

we replace $s \longrightarrow JW$

Given o.L.T.F $GH(s)$ to get Bode diagram

① Replace $s \longrightarrow J\omega$

$$GH(J\omega) = GH(s)\Big|_{s \to J\omega}$$

② $\underbrace{|GH(J\omega)|}_{\text{magnitude}}$, $\underbrace{\angle GH(J\omega)}_{\text{Angle}}$

③ $$|GH(J\omega)|_{dB} = 20 \log |GH(J\omega)|$$

dB → ديسيبل

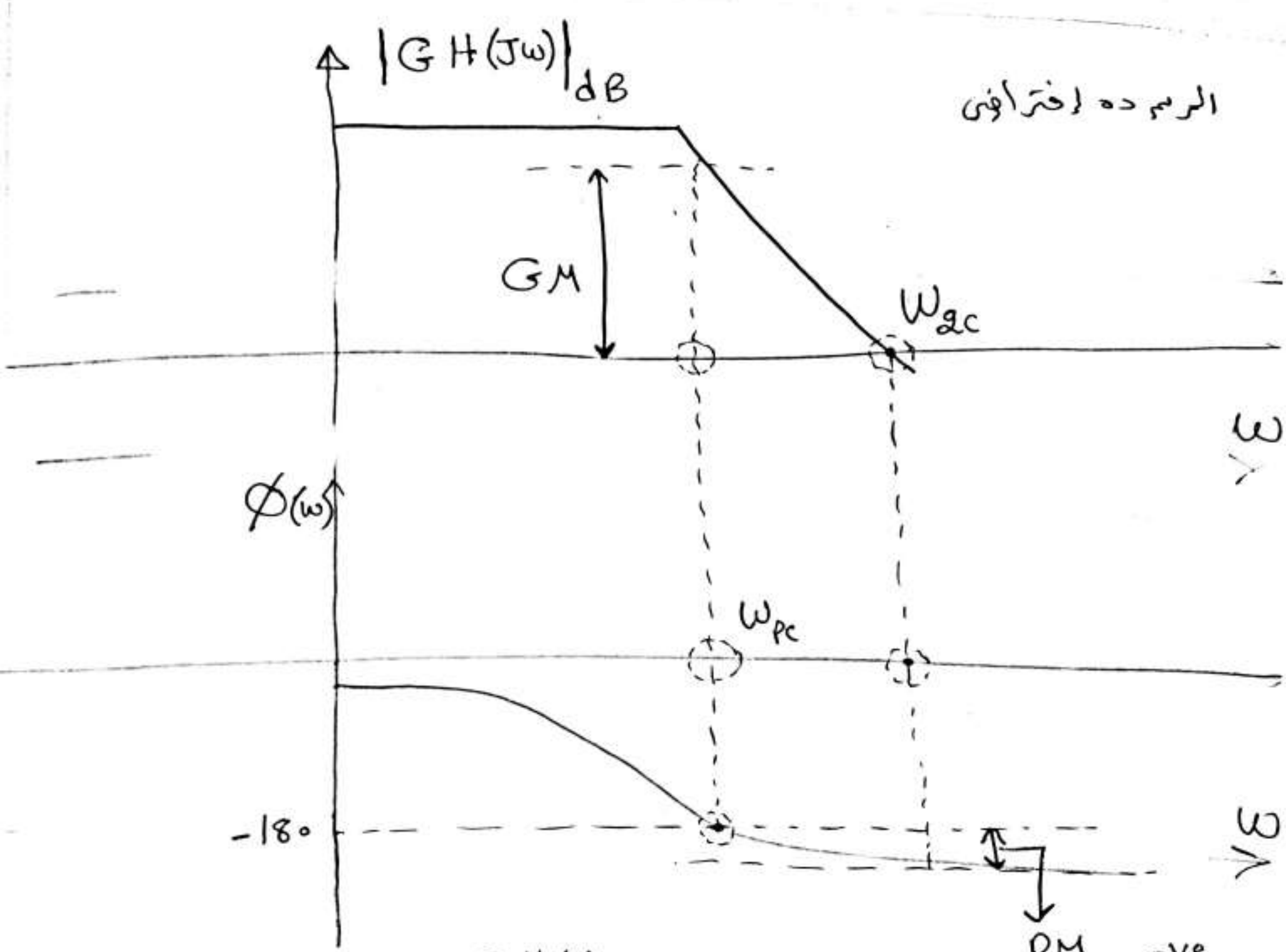
$\boxed{2}$ Lec 1

لأن الرسمة هتكون كبيرة فلازم تصغرها .

$$|\quad| = \sqrt{(Real)^2 + (img.)^2}$$

$$\phi = \tan^{-1}\left(\frac{imag.}{Real}\right)$$

الرسم ده إفتراضى



<u>ch. eq</u> $1 + GH(s) = 0$

$$GH(s) = -1 + J0$$

$\therefore \; |\;|_{dB} = 0$

$\angle GH(J\omega) = -180$

جاية من ال ch. eqn
هنحسب ال (stability) منها .

$\omega_{pc} \equiv$ phase cross freq.

$GM = -ve \longrightarrow$ zero dB عشان فوق ال

$\omega_{gc} \equiv$ gain cross freq.

التردد المناظر لـ gain مقداره zero dB

$PM = -ve \longrightarrow$ Phase Margin

لأنها تحت الزاوية 180°

stability

① GM & PM

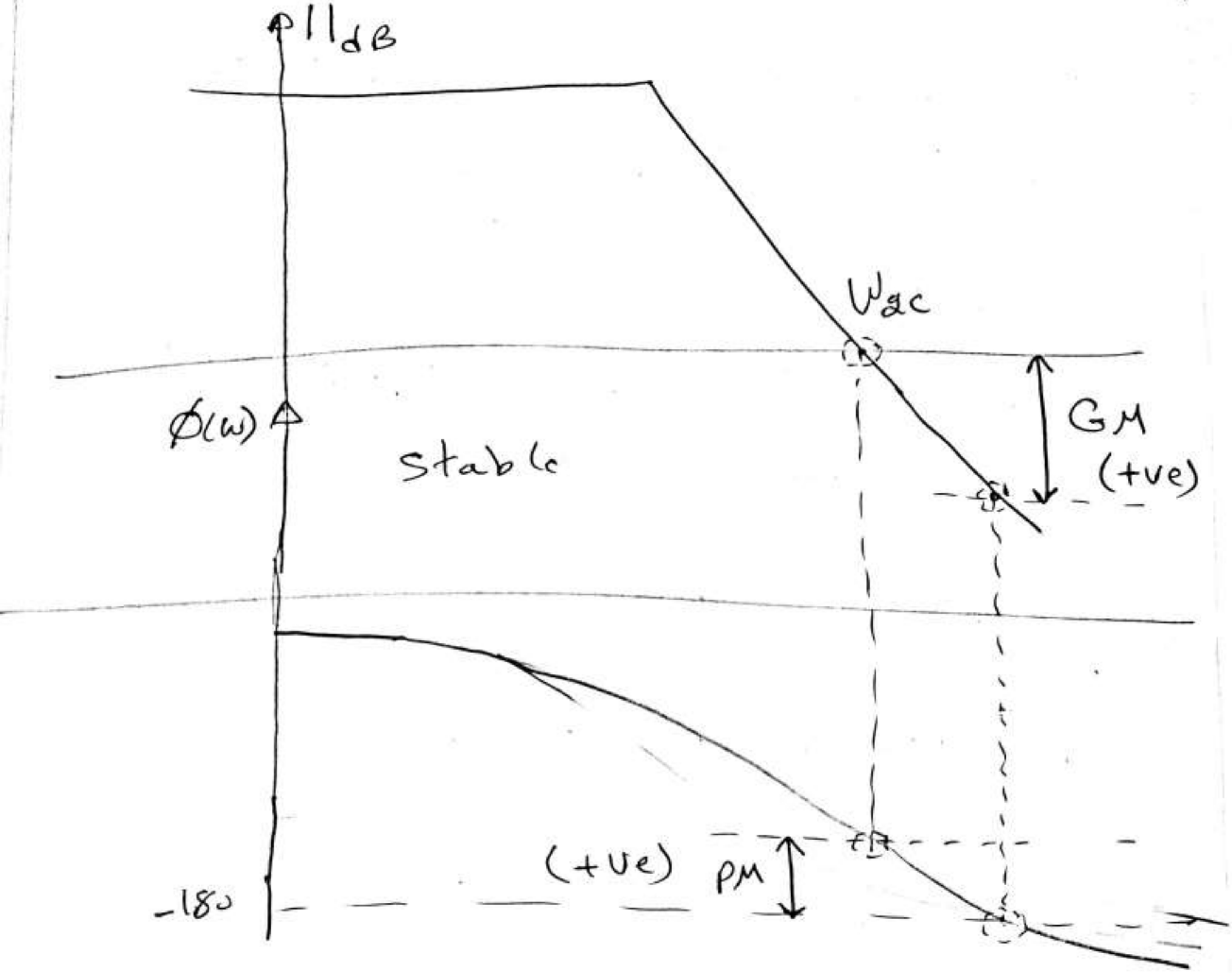
a) > 0 $(+ve) \longrightarrow$ stable

b) < 0 $(-ve) \longrightarrow$ unstable

c) $= 0$ critical stable

Note

$$|GH(j\omega)|_{dB} = 20 \log |GH(j\omega)|$$



$$\boxed{PM = 180 + \phi(\omega)\Big|_{\omega=\omega_{gc}}}$$

* The open loop T.F $GH(s)$ can be in the following forms:-

① $GH(s) = K$

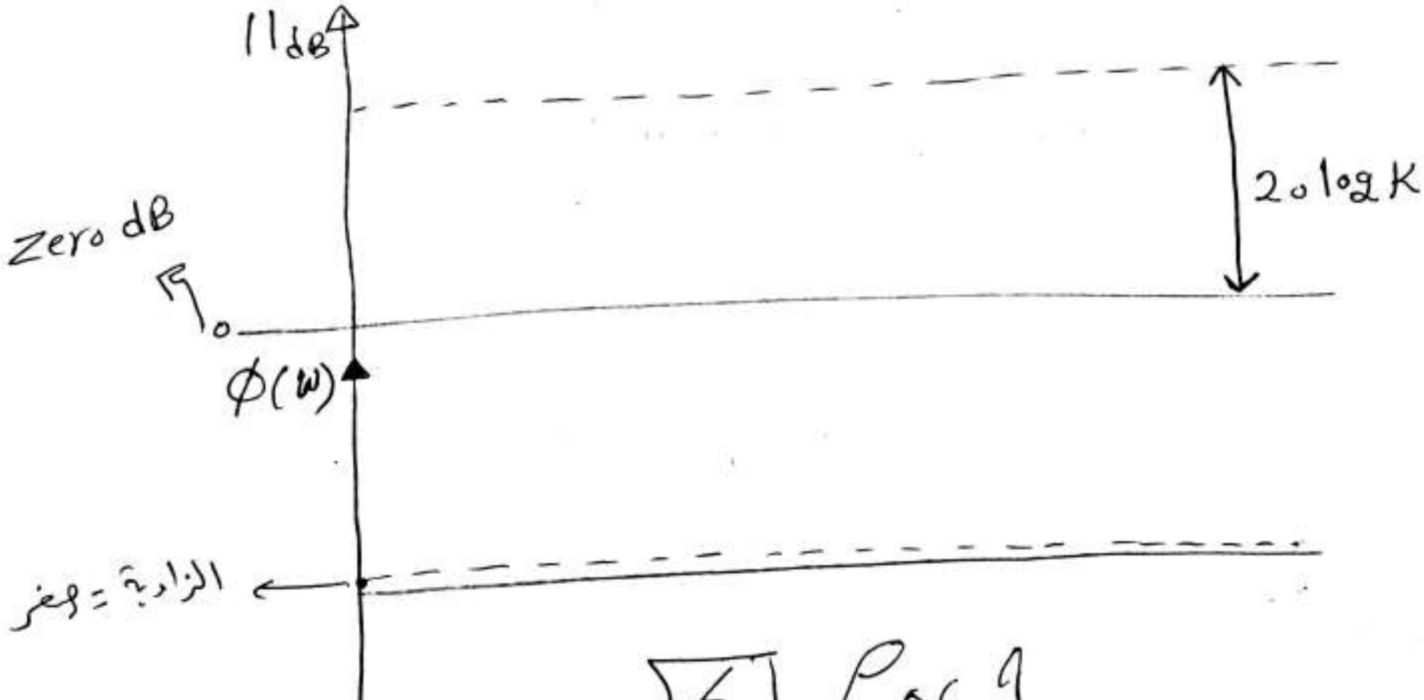
a) $s \rightarrow J\omega$ $\qquad GH(s) = GH(J\omega) = K$

ثابت ملوش دعوة بأي تغيير

b) $|GH(J\omega)| = K$

c) $|GH(J\omega)|_{dB} = 20 \log K$

d) $\phi(w) = \tan^{-1}\left(\frac{0}{K}\right) = 0$



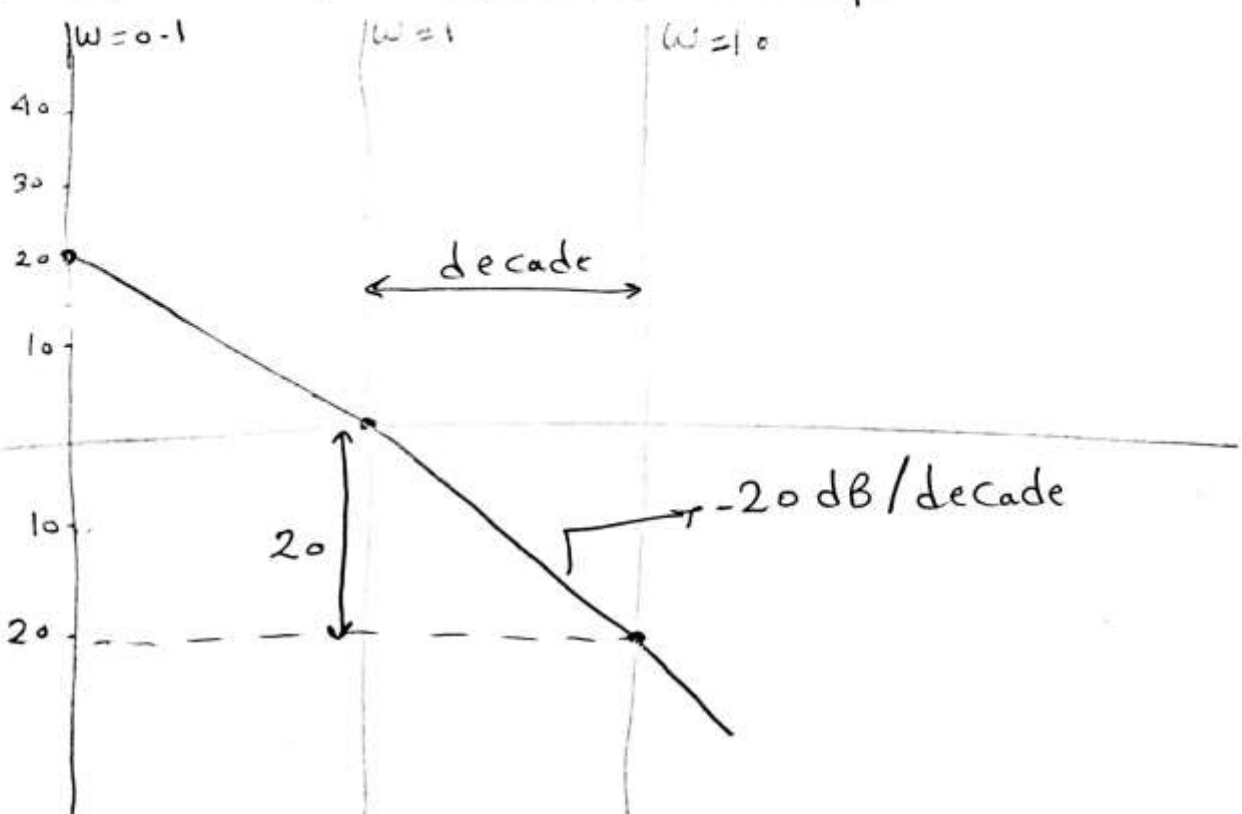
(2) $GH(s) = \frac{1}{s}$

a) $s \to J\omega \Rightarrow GH(J\omega) = \frac{1}{J\omega}$

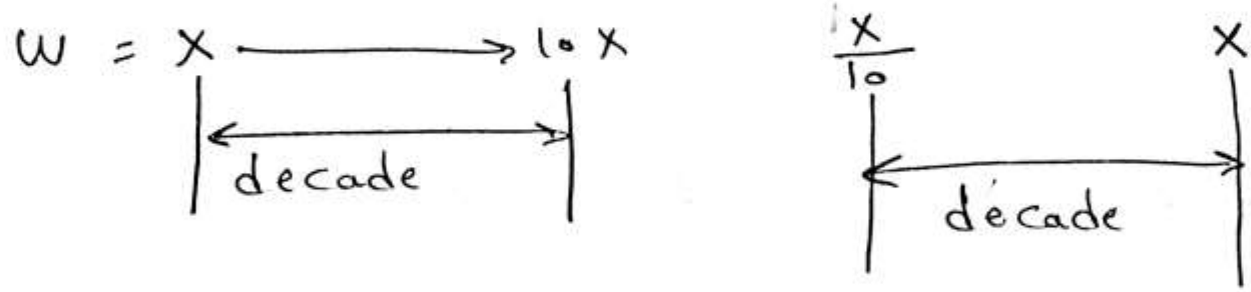
b) $|GH(J\omega)| = \frac{1}{\omega}$ (c) $|GH(J\omega)|_{dB} = 20 \log\left(\frac{1}{\omega}\right)$

$\Rightarrow |GH(J\omega)|_{dB} = 20 \log \omega^{-1} = -20 \log \omega$

ω	0.1	1	10	100
$\| \|_{dB}$	20	0	-20	-40

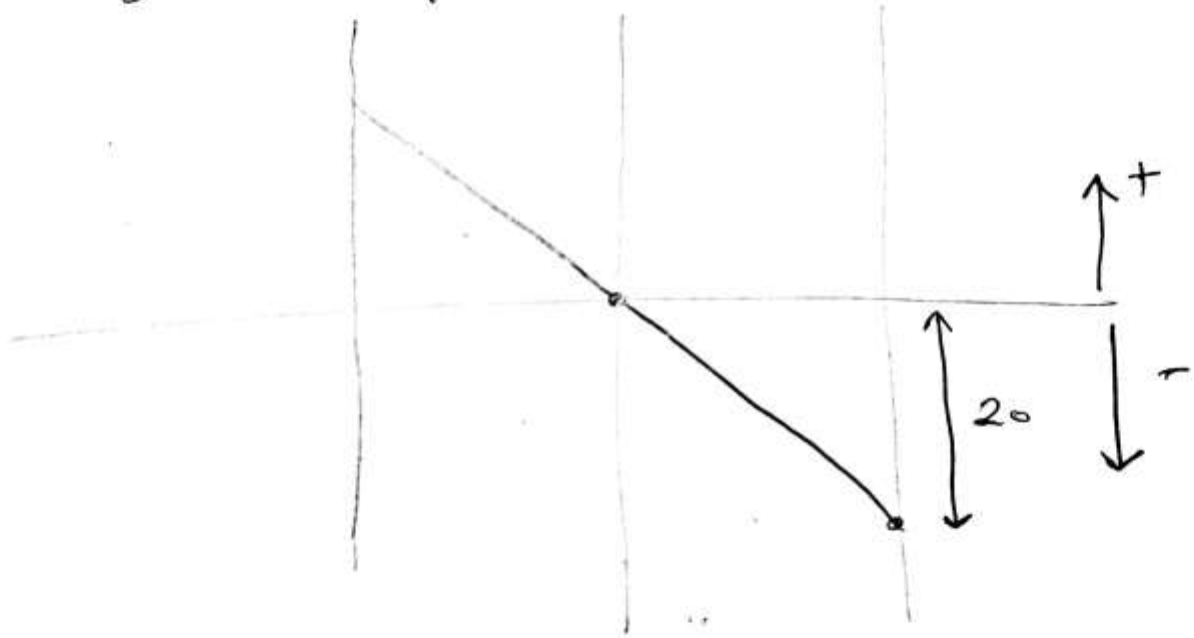


$\phi(\omega) = -90^\circ$



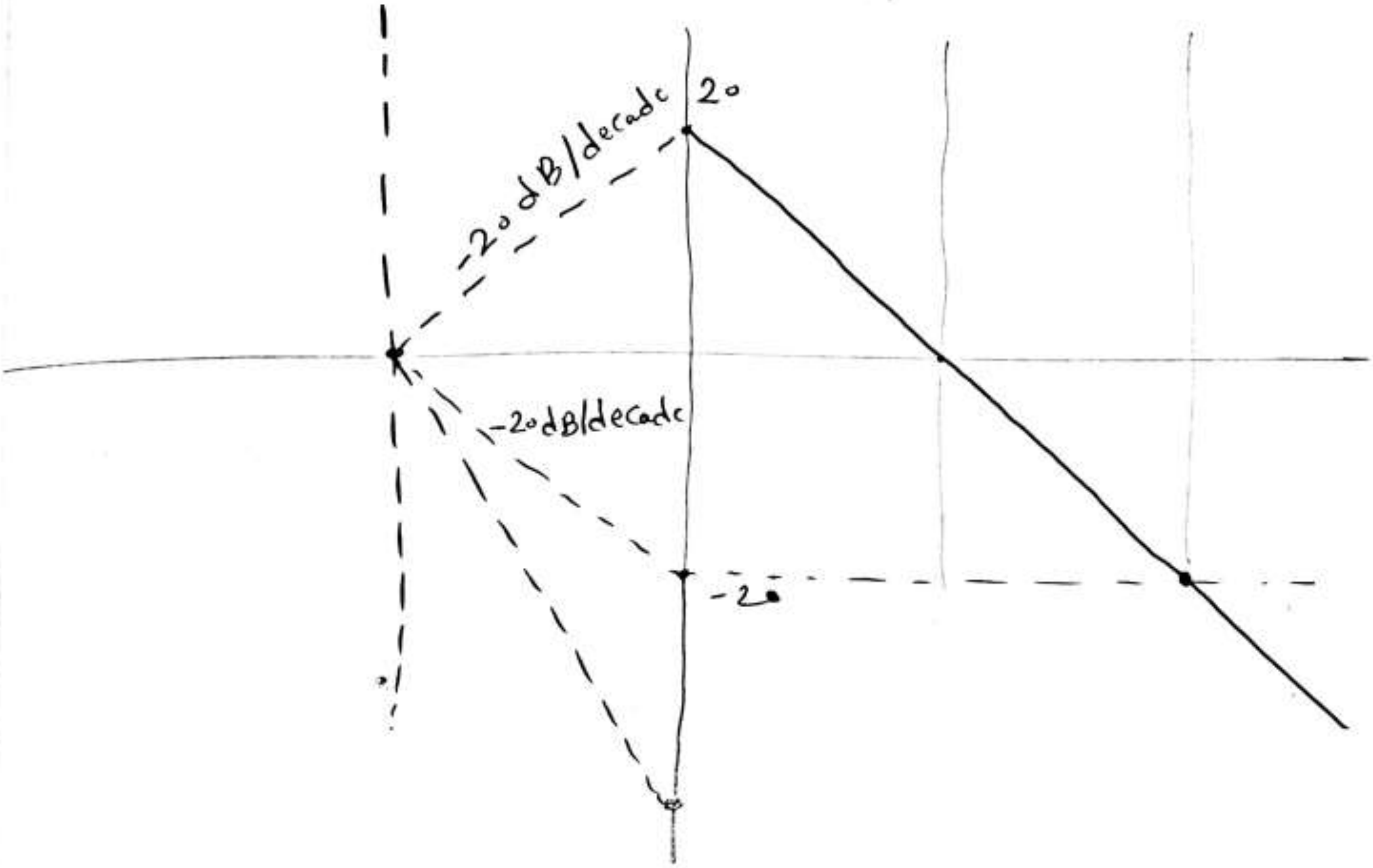
← في ورقة ال (semi log) فوقه بيقسم مربعات كبيرة
كل مربع عبارة عن (decade) وتبدأ من عند (ω=value)
مش هتبدأ عند $\omega=0$.

← لو الحالة دي جاتلك عندك خط ميله -20 dB/decade
⟸ تعرف انه خط مستقيم لو عرفت عليه نقطة تجيب الباقي.



← خد (decade) يمين وآخر شمال
← الميل بالسالب انزل لتحت بالقيمة 20 ولو الموجب تطلع لفوق.

ـــه أو حل آخر (إنك تعمل (decade) هامش.



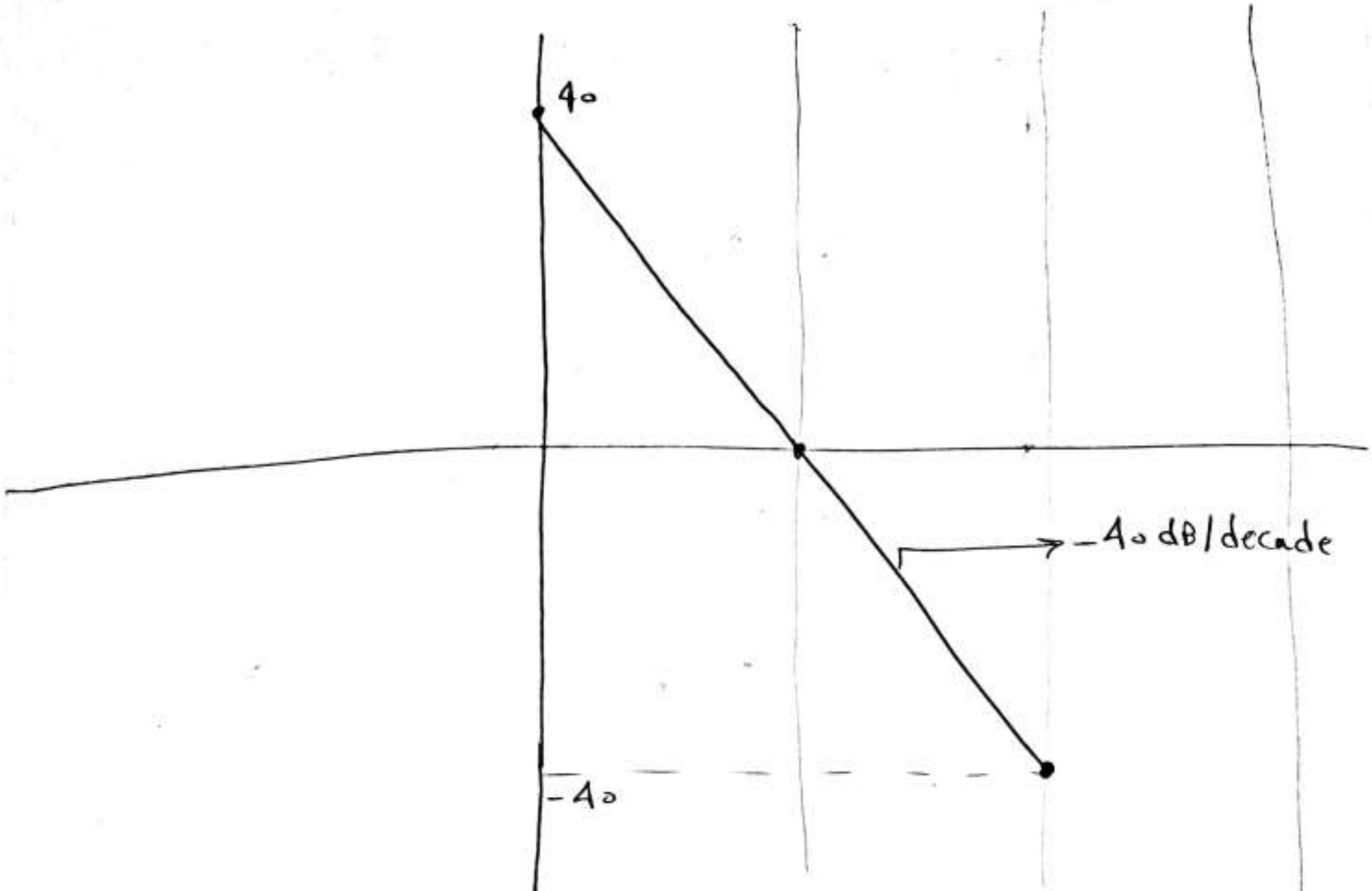
③ $GH(s) = \frac{1}{s^2}$

a) $s \longrightarrow J\omega$ $\qquad GH(J\omega) = \frac{1}{(J\omega)^2}$

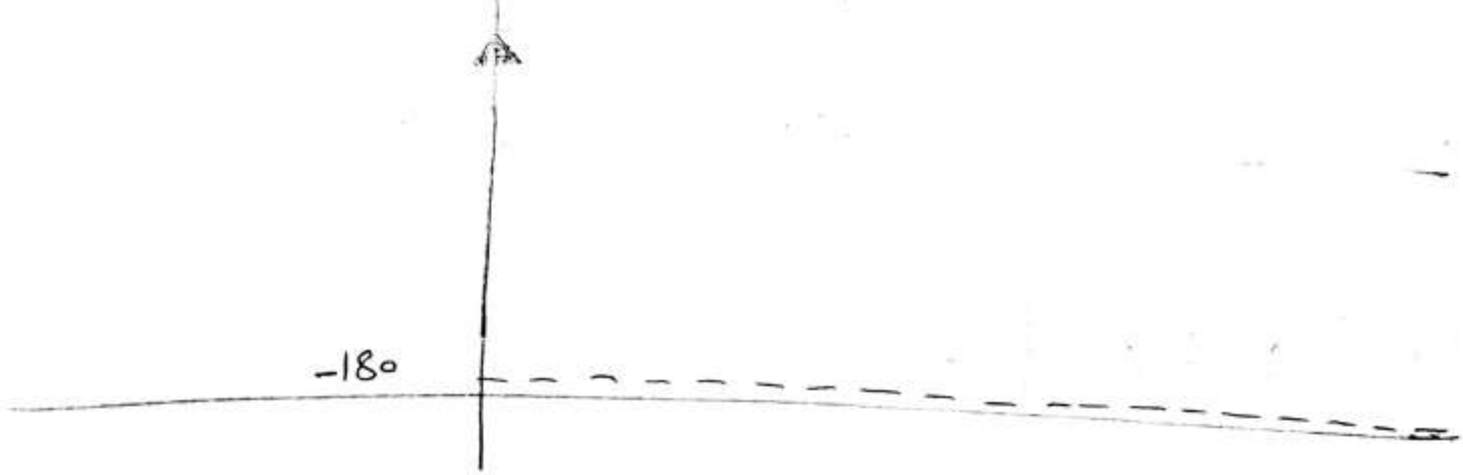
b) $|GH(J\omega)| = \frac{1}{\omega^2}$

c) $| \quad |_{dB} = 20 \log \frac{1}{\omega^2} = 20 \log \omega^{-2}$

$= -40 \log \omega$



$$\frac{1}{s^2} \Rightarrow \phi(\omega) = -180$$



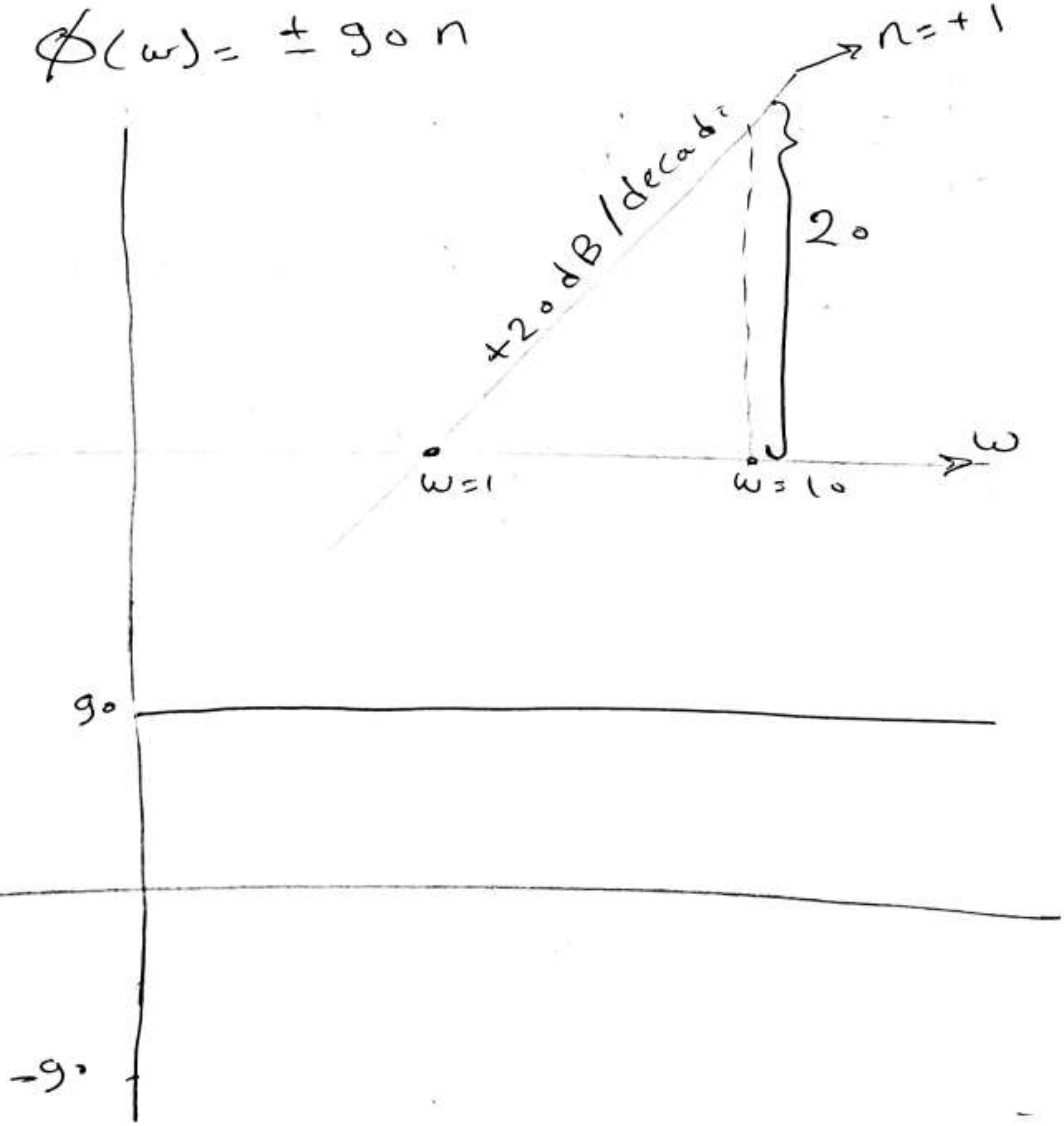
$$GH(s) = (s)^{\pm n}$$

$| \, |_{dB} \Rightarrow$ خط مستقيم يمر د $\omega = 1$ ميله

البسط $\longrightarrow$ $\pm 20n$ dB/decade

المقام

$$\phi(\omega) = \pm 90n$$



$\boxed{9}$ $GH(s) = \frac{K}{s}$

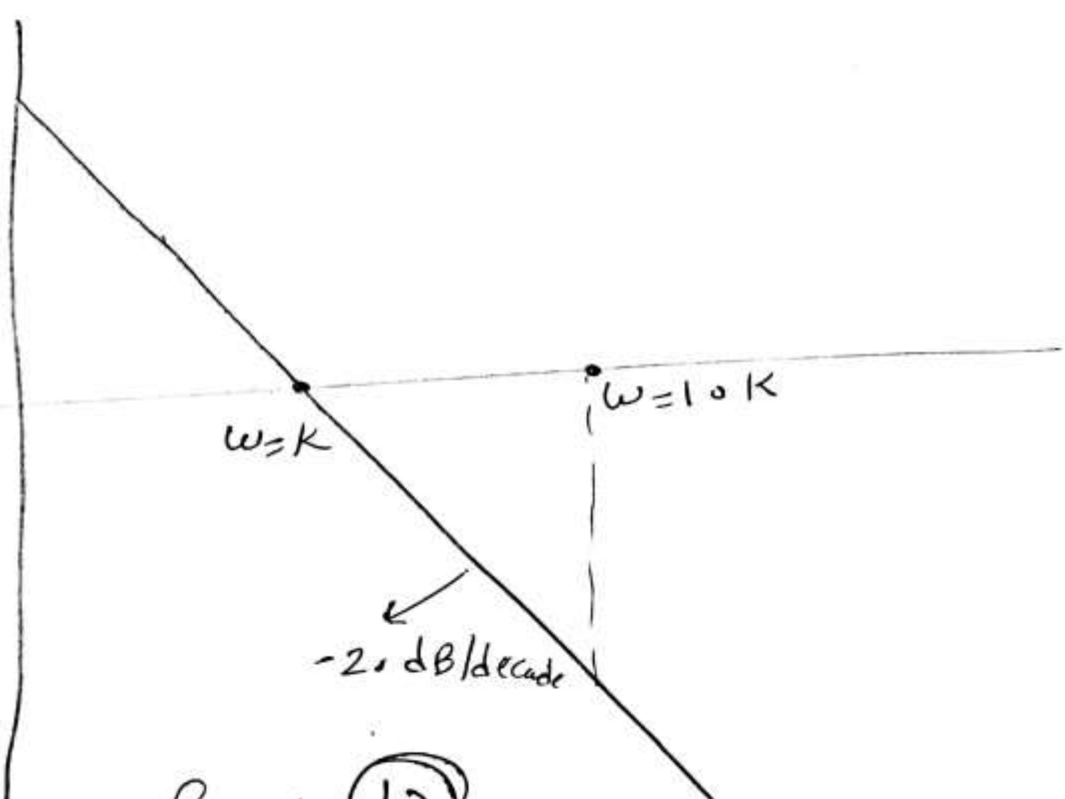
$s \rightarrow j\omega \qquad GH(j\omega) = \frac{K}{j\omega}$

$$|GH(j\omega)| = \frac{K}{\omega}$$

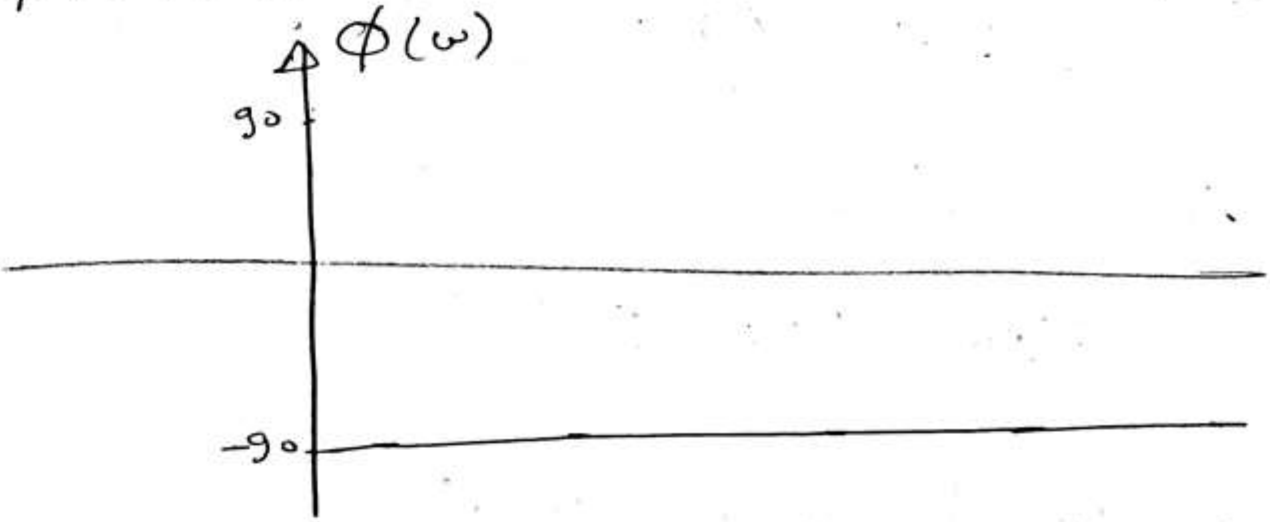
$$|GH(j\omega)|_{dB} = 20 \log\left(\frac{K}{\omega}\right)$$

$$= 20 \log\left(\frac{\omega}{K}\right)^{-1} = -20 \log\left(\frac{\omega}{K}\right)$$

← نقطة مرور الخط هو مازال خط مستقيم بنفس الميل
لكنه يمر بالنقطة $\omega = K$



$\phi(\omega) = -90$



$* \quad GH(s) = \dfrac{K}{s^2}$

ـــ خط مستقيم يمر $\omega = \sqrt{K}$ ميله $= -40$ dB/decade

$\phi(\omega) = -18\mathring{0}$

5 $* \quad GH(s) = \dfrac{K}{s^3}$

ـــ خط مستقيم يمر بالنقطة $\omega = \sqrt[3]{K}$ ميله

-60 dB/decade ، $\phi(\omega) = -27\mathring{0}$

$\boxed{6}$

$$GH(s) = \left(1 + \frac{s}{c}\right) \quad , c \rightarrow \text{constant}$$

a) $s \longrightarrow J\omega$

$$GH(J\omega) = \left(1 + J\frac{\omega}{c}\right)$$

$$|GH(J\omega)| = \sqrt{1 + \left(\frac{\omega}{c}\right)^2}$$

$$\phi(\omega) = \tan^{-1}\left(\frac{\omega}{c}\right)$$

$$|GH(J\omega)|_{dB} = 20 \log \sqrt{\left(1 + \frac{\omega}{c}\right)^2}$$

approximation

① $\omega < c \implies \left(\frac{\omega}{c}\right)^2 \ll 1$ (neglect)

$$|GH(J\omega)|_{dB} = 20 \log \sqrt{1+0} = 0$$

② $\omega > c \implies \left(\frac{\omega}{c}\right)^2 \gg 1 \quad \therefore (1 \rightarrow \text{neglect})$

$$|GH(J\omega)|_{dB} = 20 \log \sqrt{1 + \left(\frac{\omega}{c}\right)^2}$$

$$= 20 \log \frac{\omega}{c}$$

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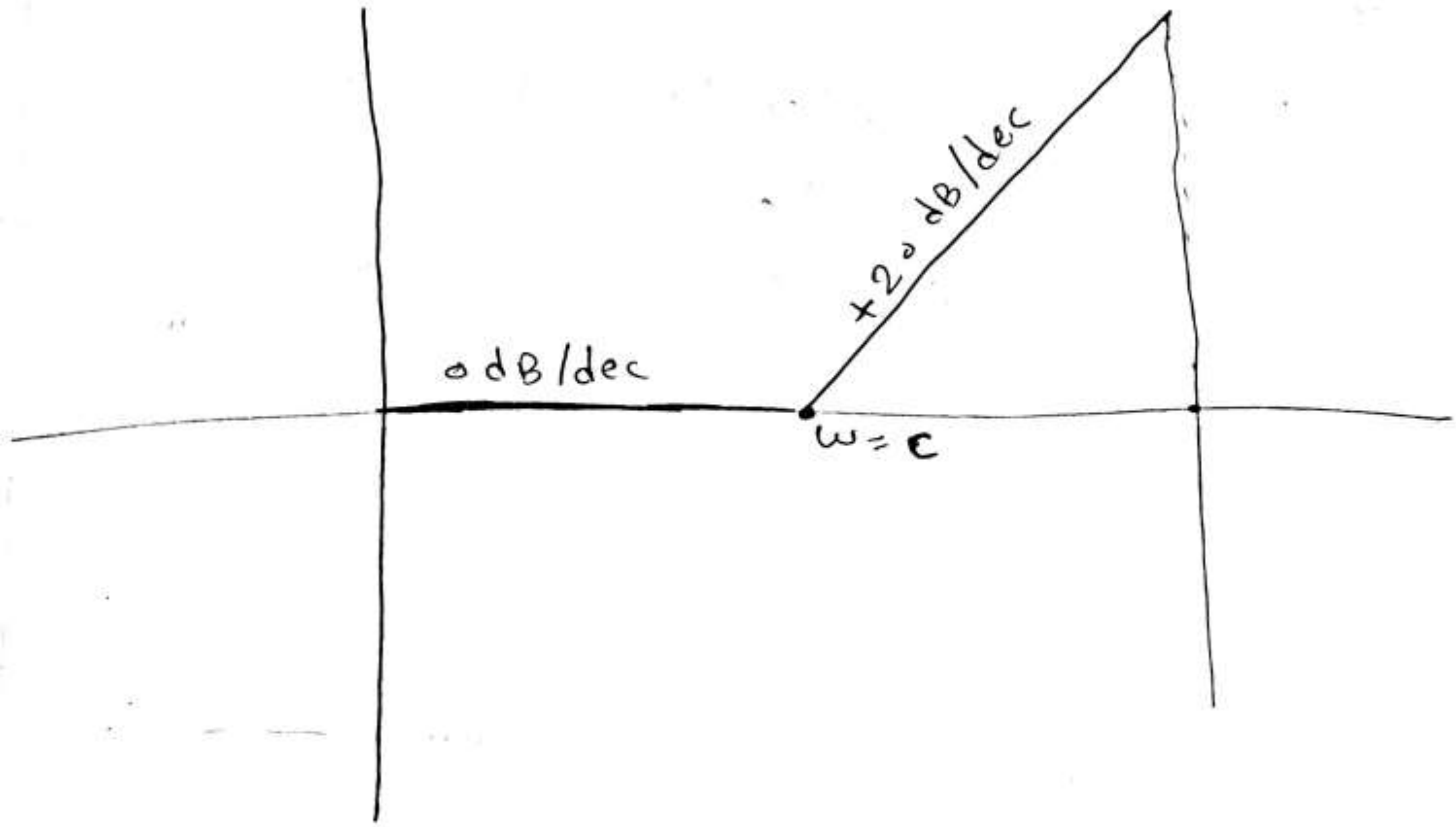
$$|GH(J\omega)|_{dB} = 20 \log \sqrt{1 + \left(\frac{\omega}{c}\right)^2}$$

$$= 20 \log \frac{\omega}{c}$$

$\Rightarrow 20 \log\left(\frac{\omega}{c}\right)$

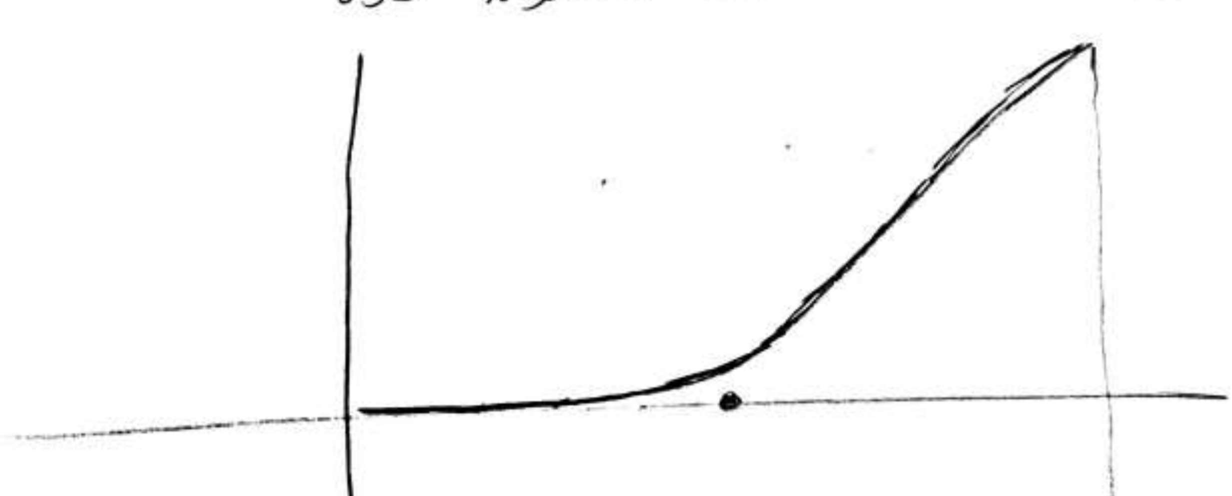
ــــ خط مستقيم يمر بالنقطة $\omega = c$



$c \rightarrow$ corner frequency

ــــ نقطة إنقلاب الميل يعني الميل قبلها بقيمة وبعدها بقيمة تانية.

ــــ لو بتحل exact هتكون كده

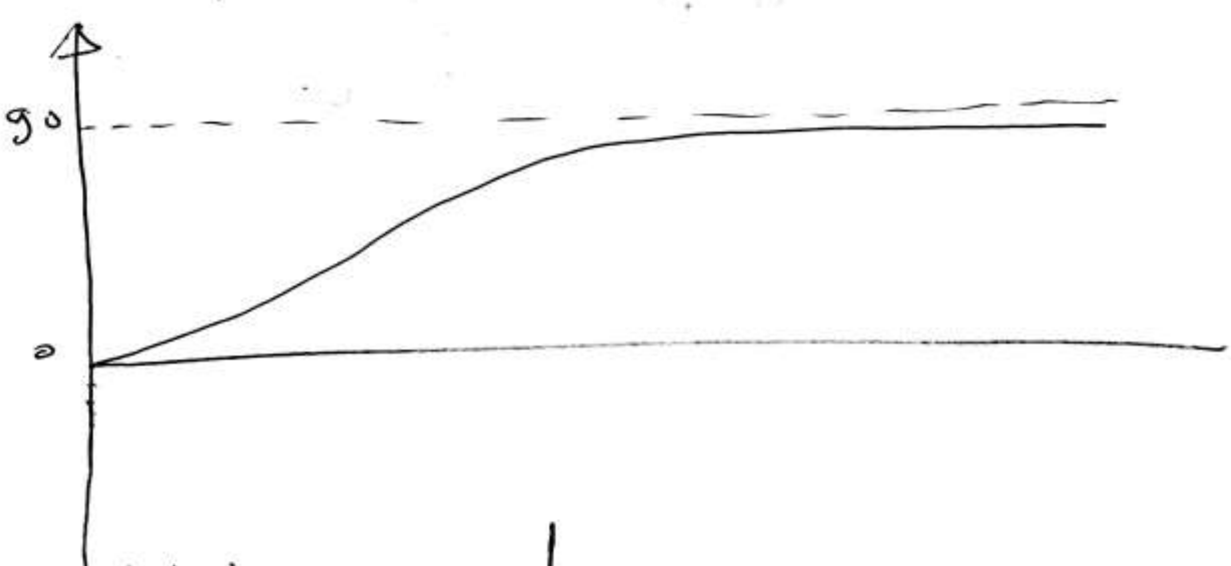


$$\phi(\omega) = \tan^{-1}\left(\frac{\omega}{c}\right)$$

<u>Range</u> $0 \longrightarrow 90$

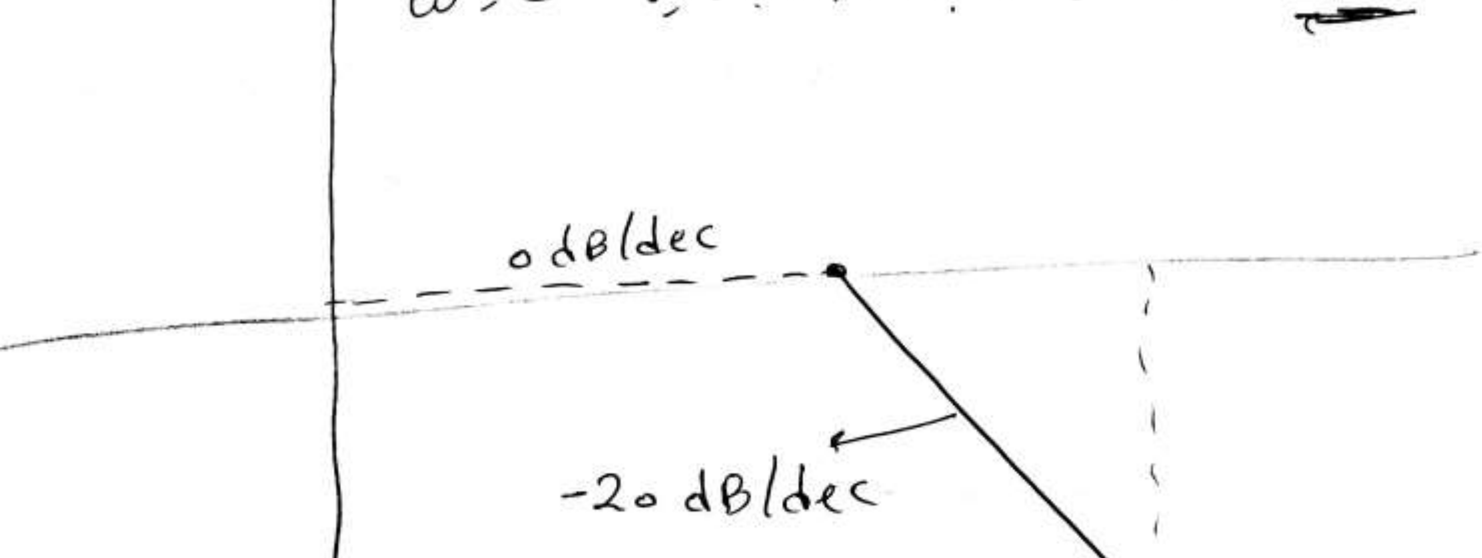
← لو بترسم (exact) هتعمل جدول

ω	0	∞
$\phi(\omega)$	0	90



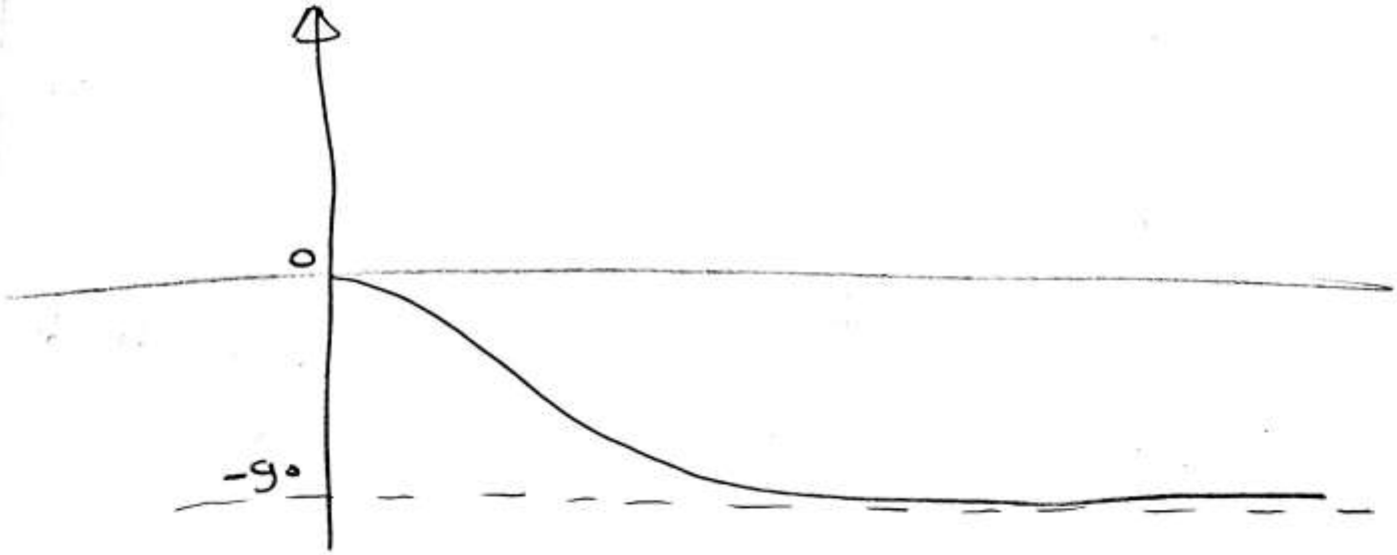
$$* \quad GH(s) = \frac{1}{1+\frac{s}{c}}$$

← الميل هيبقى بالسالب بعد قيمة $\omega = c$



$$\phi(\omega) = 0 - \tan^{-1}\left(\frac{\omega}{c}\right)$$

Range of Angle $0 \longrightarrow -90$



$$\left(1+\frac{s}{c}\right)^{\pm n} = GH(s) \quad \cancel{\pm 20n \ dB/dec}$$

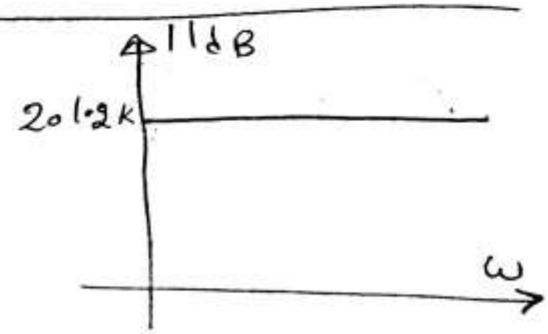
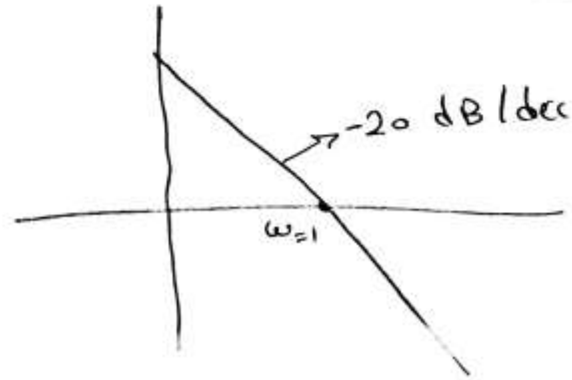
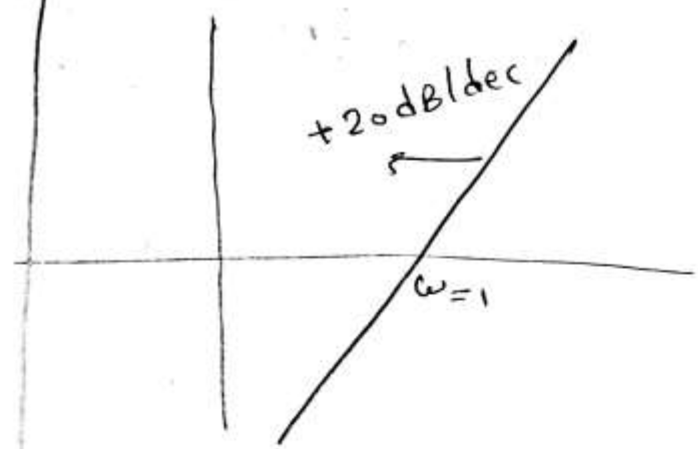
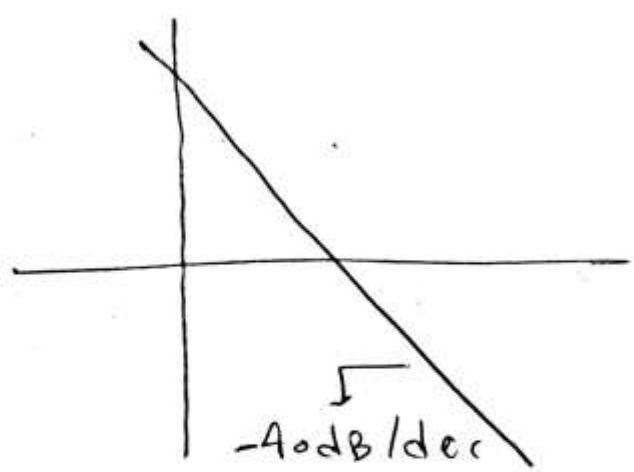
$c \Rightarrow$ Corner frequency

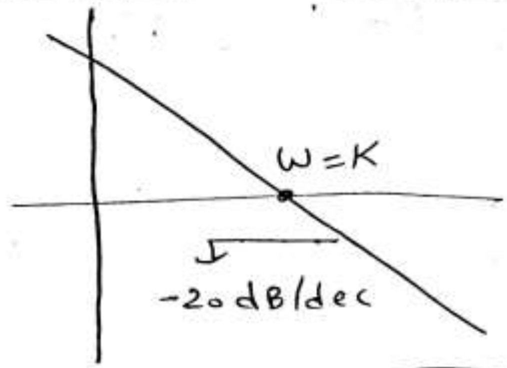
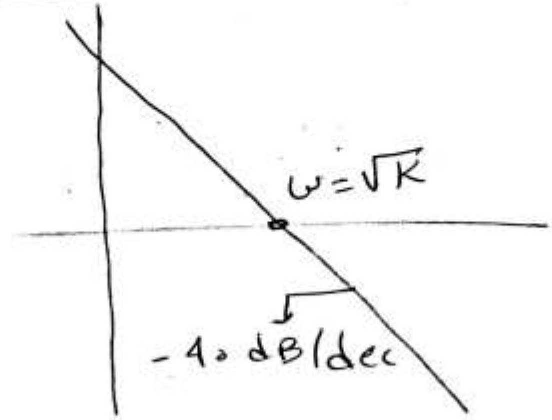
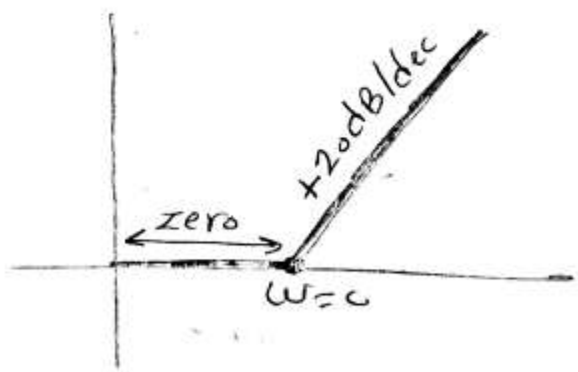
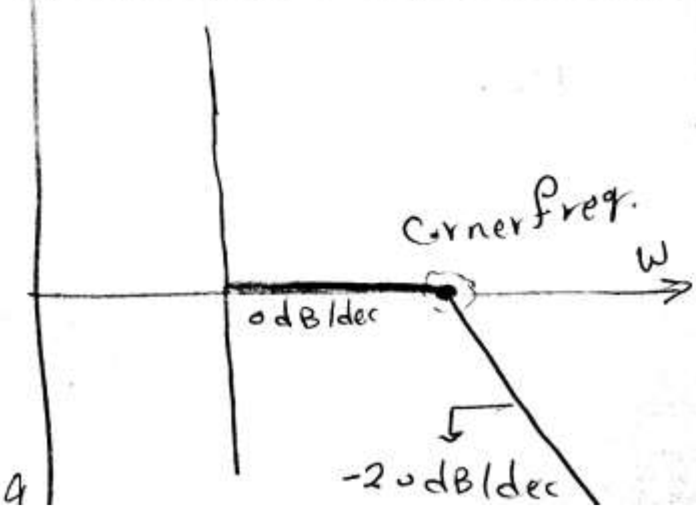
$0 \, dB/dec = (\omega = c)$ الميل قبل

$\pm 20 \, dB/dec = (\omega = c)$ بعد "

$$\phi(\omega) = \pm n \tan^{-1}\left(\frac{\omega}{c}\right)$$

Range of $\phi(\omega)$:
- $\rightarrow \omega = 0 \Rightarrow 0$
- $\rightarrow \omega = \infty \Rightarrow \pm 90 n$

Term	$\phi(\omega)$	$\lvert\ \rvert_{dB}$
K	0	
$\frac{1}{s}$ or $\frac{1}{J\omega}$	-90	
s or $J\omega$	$+90$	
$\frac{1}{s^2} \Rightarrow \frac{1}{J\omega . J\omega}$	-180	

Term	$\phi(\omega)$	$\| \ \|_{dB}$
$\frac{K}{S} \Rightarrow \frac{K}{J\omega}$	-90	
$\frac{K}{S^2} \Rightarrow \frac{K}{J\omega \cdot J\omega}$	-180	
$1+\frac{S}{c}$ $\Downarrow$ $1+J\frac{\omega}{c}$	$\tan^{-1}\left(\frac{\omega}{c}\right)$	
$\frac{1}{1+\frac{S}{c}}$ $\Downarrow$ $\frac{1}{1+J\frac{\omega}{c}}$	$-\tan^{-1}\frac{\omega}{c}$	

$|G|$ for a

$\boxed{Ex}$

$$GH(s) = \frac{10}{(1+s)(1+0.1s)}$$

* Draw the bode diagram & find GM & PM

Sol

$s \rightarrow J\omega$

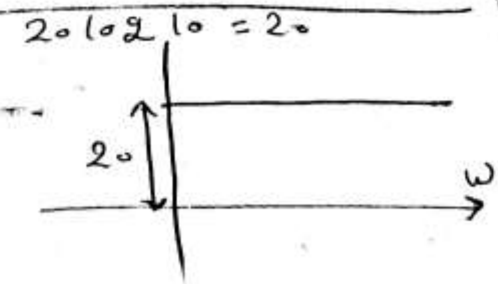
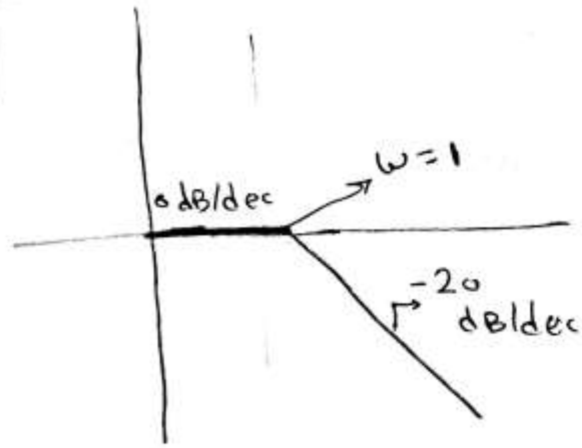
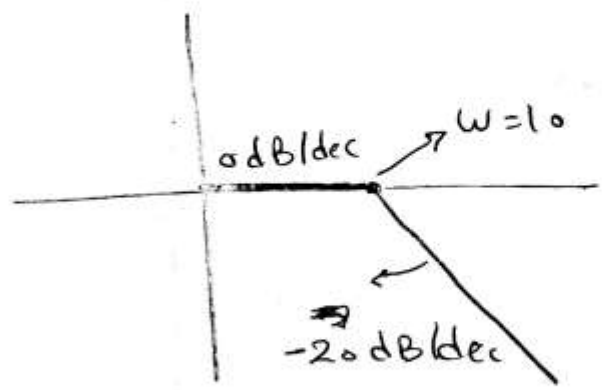
$$GH(J\omega) = \frac{10}{(1+J\omega)(1+0.1J\omega)}$$

$$= \frac{10}{\left(1+J\frac{\omega}{1}\right)\left(1+J\frac{\omega}{10}\right)}$$

$$\boxed{\phi(\omega) = \underbrace{0}_{\text{البسط}} - \underbrace{\tan^{-1}(\omega)}_{1+J\frac{\omega}{1}} - \underbrace{\tan^{-1}\left(\frac{\omega}{10}\right)}_{1+J\frac{\omega}{10}}}$$

~~For~~

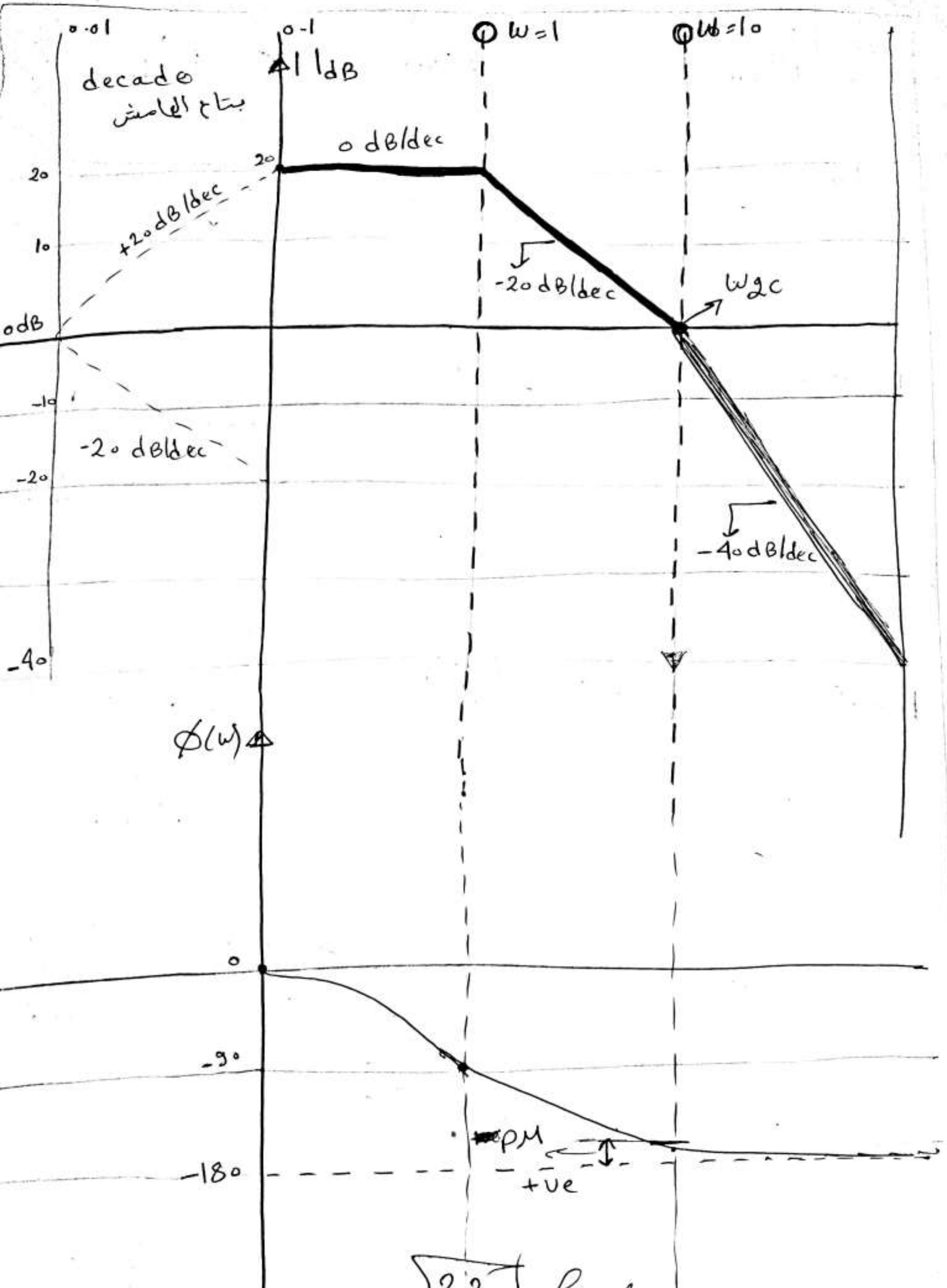
$\boxed{20}$ Lec 4

Term	$\phi(\omega)$	$\lvert\ \rvert_{dB}$
10	0	
$1 + J\frac{\omega}{1}$	$-\tan^{-1}(\omega)$	
$1 + J\frac{\omega}{10}$	$-\tan^{-1}\left(\frac{\omega}{10}\right)$ ⋮	

$$\therefore \phi(\omega) = 0 - \tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{10}\right)$$

زى ما قلنا
فى الصفحة
السابقة

$\boxed{21}$ Lec4


0.01
0.1
ω=1
ω=10
decade
بتاع الهامش
| |dB
0 dB/dec
20
10
0dB
-10
-20
-40
+20dB/dec
-20 dB/dec
-20dB/dec
ωgc
-40dB/dec
ϕ(ω)
0
-90
-180
PM
+ve

ω	0	0.1	1	10	100	∞
$\phi(\omega)$	0	-6.3	-50.7	-129.3	-173.7°	-180

w_{gc}

$$PM = 180 + \phi(\omega = \omega_{gc}) = 180 - 129.3 = \text{—}$$

أو تجيبها من الرسم.

$GM = \infty$

← عشان لم يحدث تقاطع تحت.

$\longrightarrow$ system stable

$\boxed{23}$ Lec4

← الصفحة السابقة تم توضيح الرسم على ورق semilog.